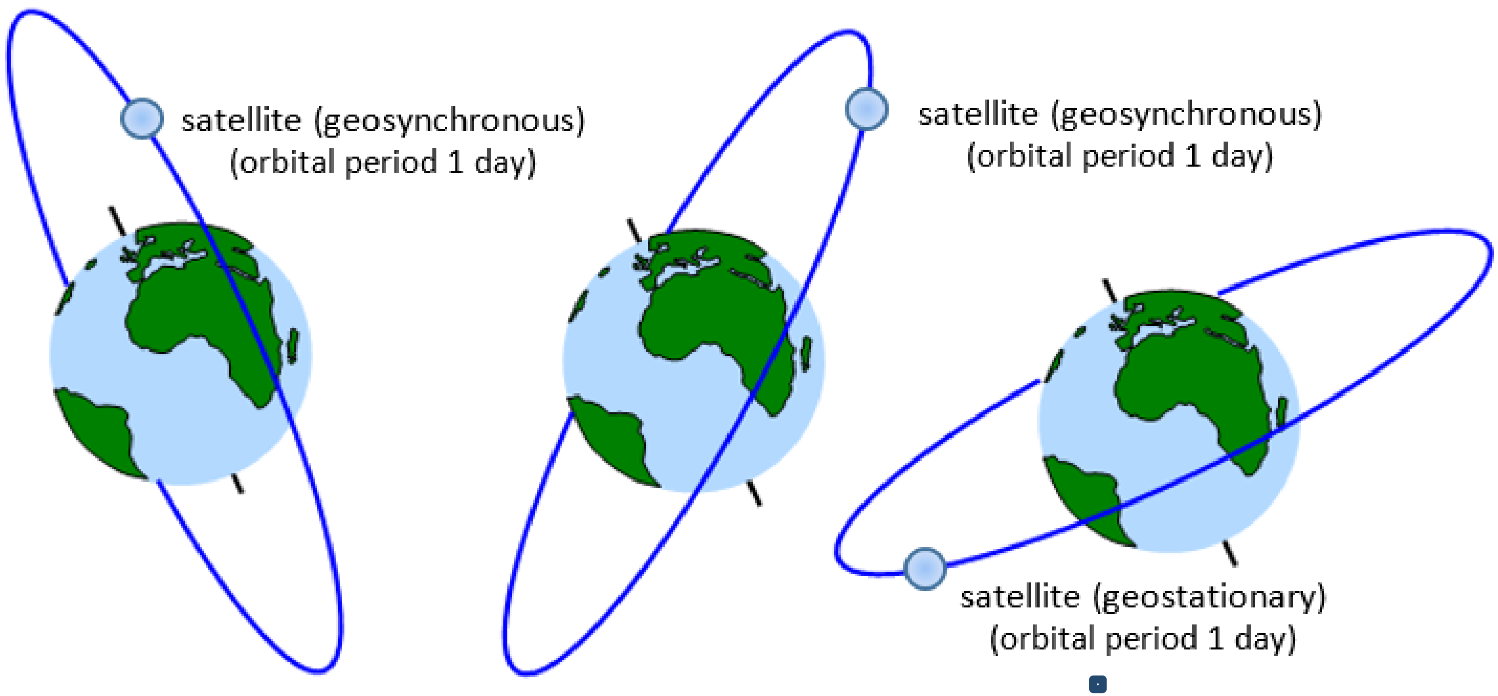
Ingegneria delle Telecomunicazioni

Satellite Communications

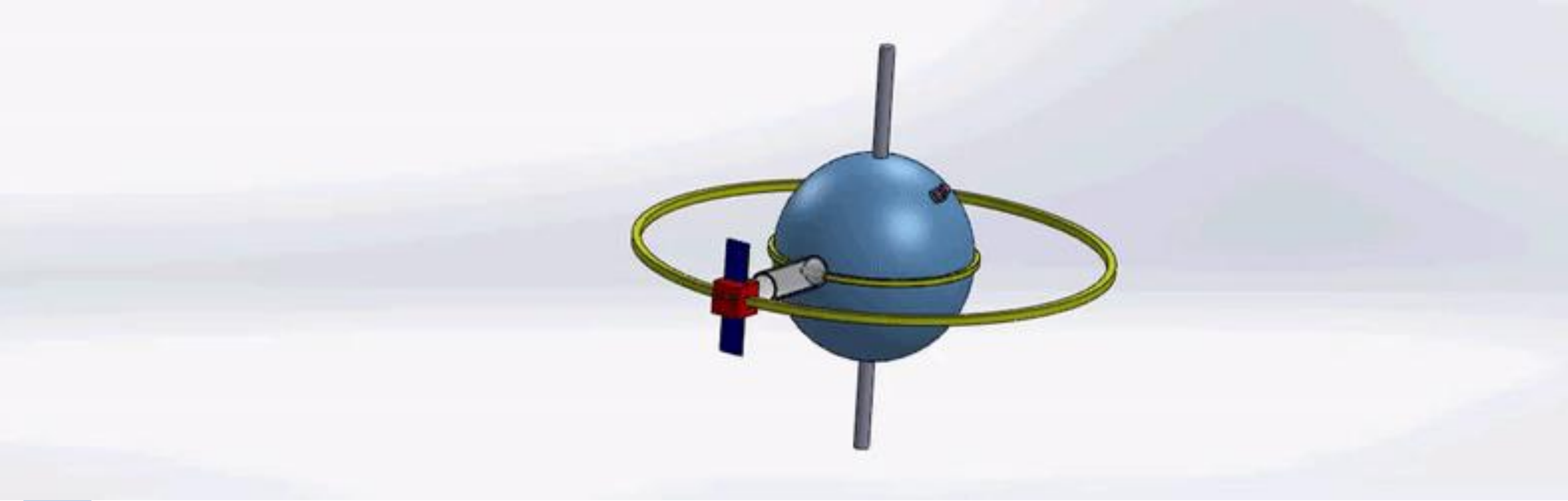
3. LEO, MEO, HEO, what else?

Marco Luise
marco.luise@unipi.it

Geosynchronous & Geostationary Orbits

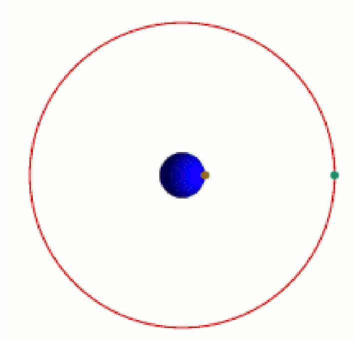


Geostationary Satellite – the MOA Communication Satellites



Computation of GEO Height

Gravity Force=Centrifugal Force

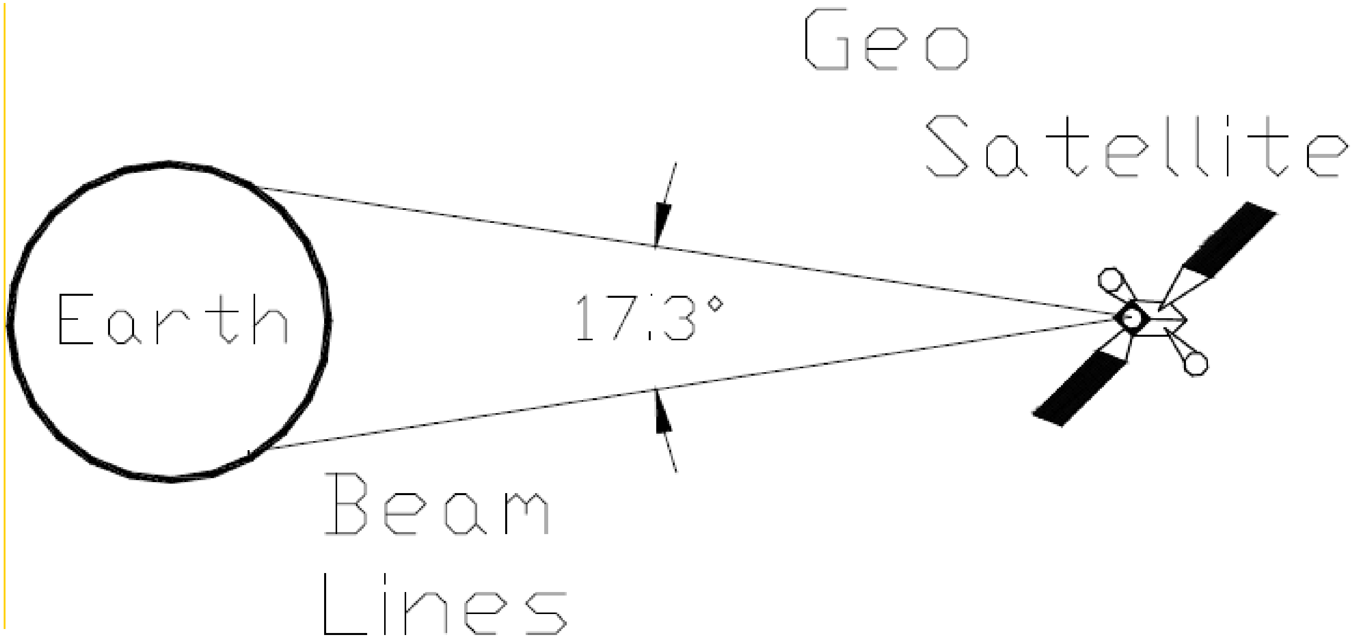


$$G\frac{mM}{r^2} = m\omega^2 r$$

- (Unknown) Satellite Height above ground *h*
- (Equatorial) Earth Radius: *R*=6,378 km
- Distance from the center of the Earth *r*=*R*+*h*
- Universal Gravitational Constant *G*=6.674 x 10^{-11} Nm^2/kg^2
- Geosynchronous angular speed $\omega = 2\pi/(24 \times 3600)$ rad/s
- Earth Mass *M*=5.97 x 10^{24} kg
- Standard gravitational parameter μ=GM= 3.99 x 10^{14} m^3s^{-2}

$$h = \sqrt[3]{\frac{GM}{\omega^2}} - R = \text{42,235-6,378=35,864 km}$$

It’s very far !



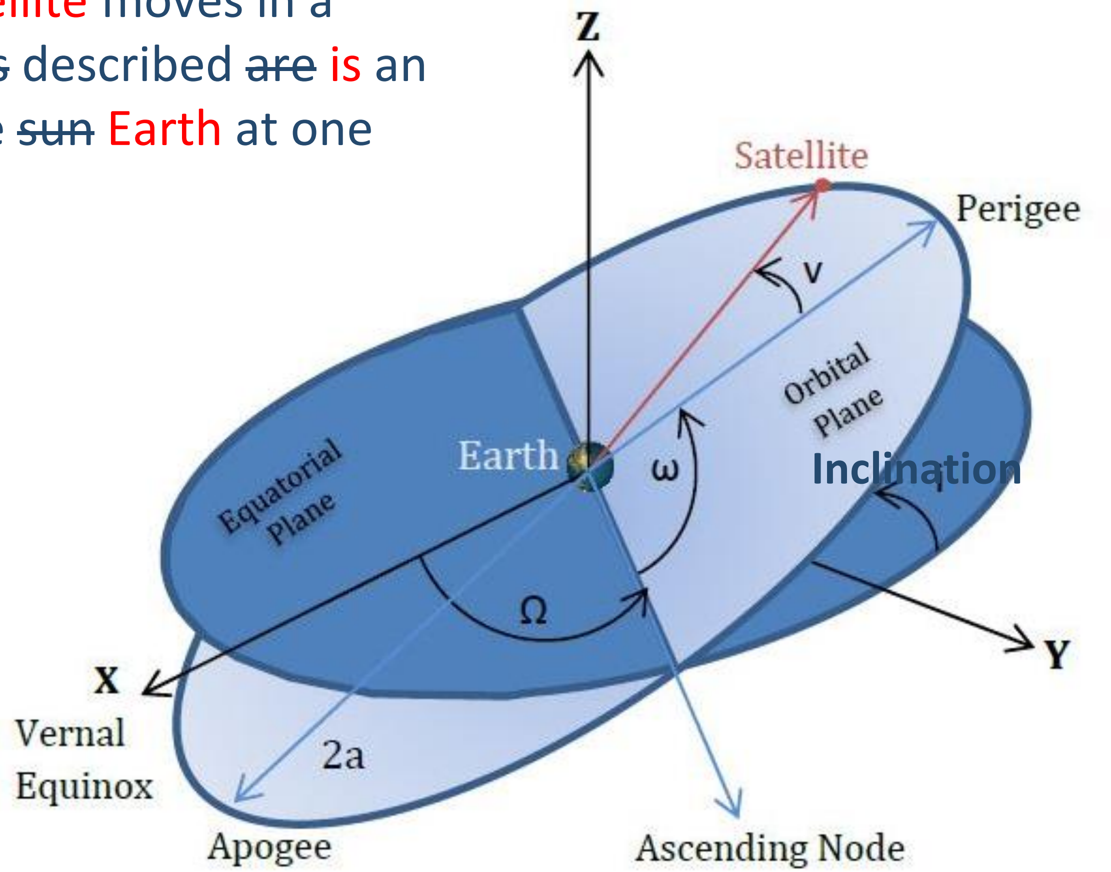
Does the satellite signal get to Earth loud enough (and vice-versa?)

Kepler’s Laws

1. The planets move in a plane; the orbits described are ellipses with the Sun at one focus (1602).
2. The vector from the Sun to the planet sweeps equal areas in equal times (the law of areas, 1605).
3. The ratio of the square of the period T of revolution of a planet around the sun to the cube of the semi-major axis a of the ellipse is the same for all planets (1618).

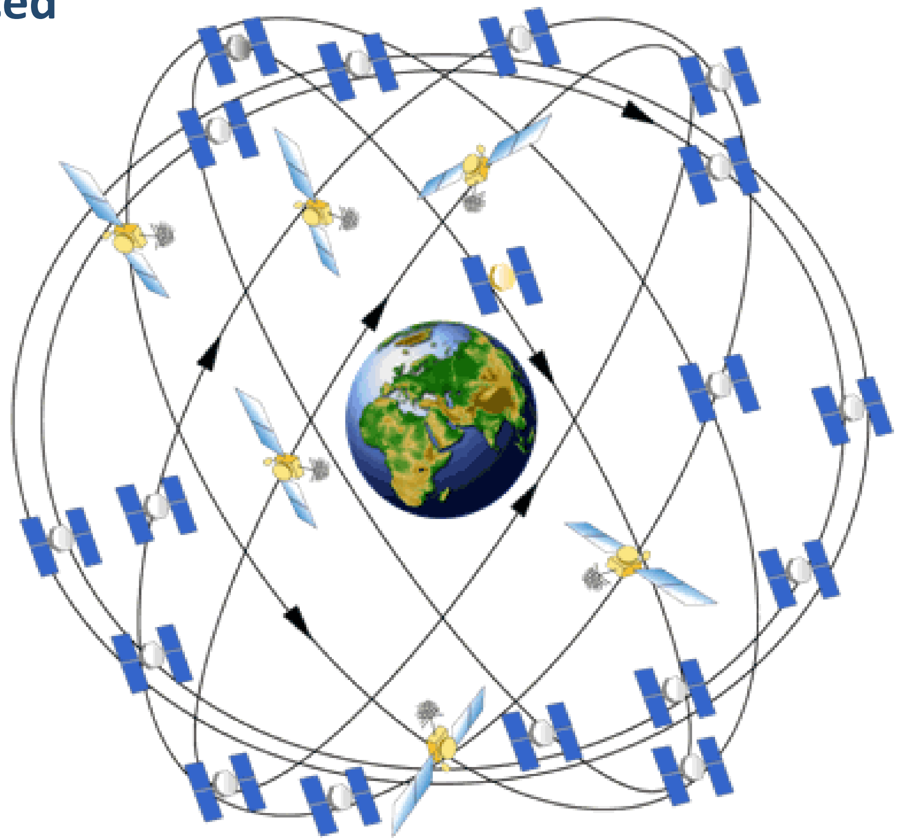
1st Law - Orbital Parameters

1. The ~~planets~~ satellite moves in a plane; the orbits described ~~are~~ is an ellipses with the ~~sun~~ Earth at one focus

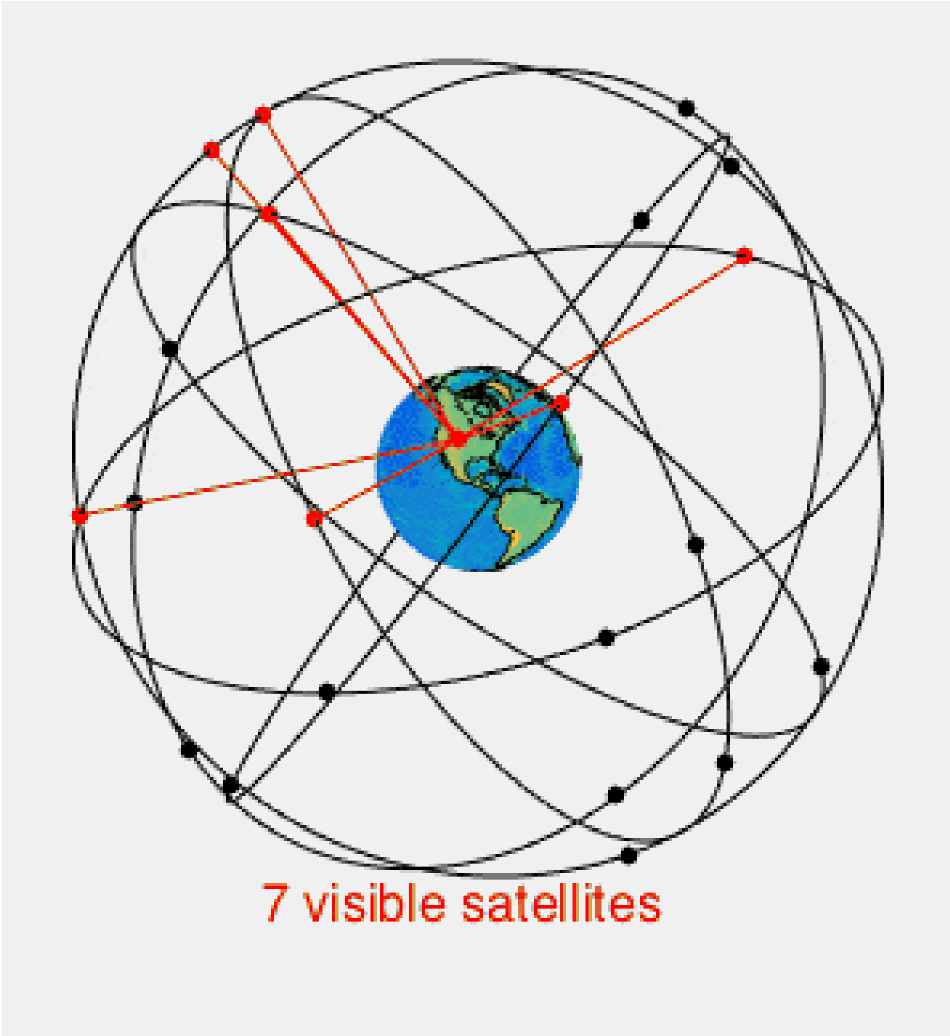


GPS Constellation

- 24 satellites
- (Almost) Circular Orbits on 6 equi-spaced planes
- Inclination of 55 degrees
- Altitude *h*=20,200 km
- Period (about) 12 hours (11h 58' 2s)
- Thie 24-slot arrangement ensures users can view at least four satellites from virtually any point of the planet

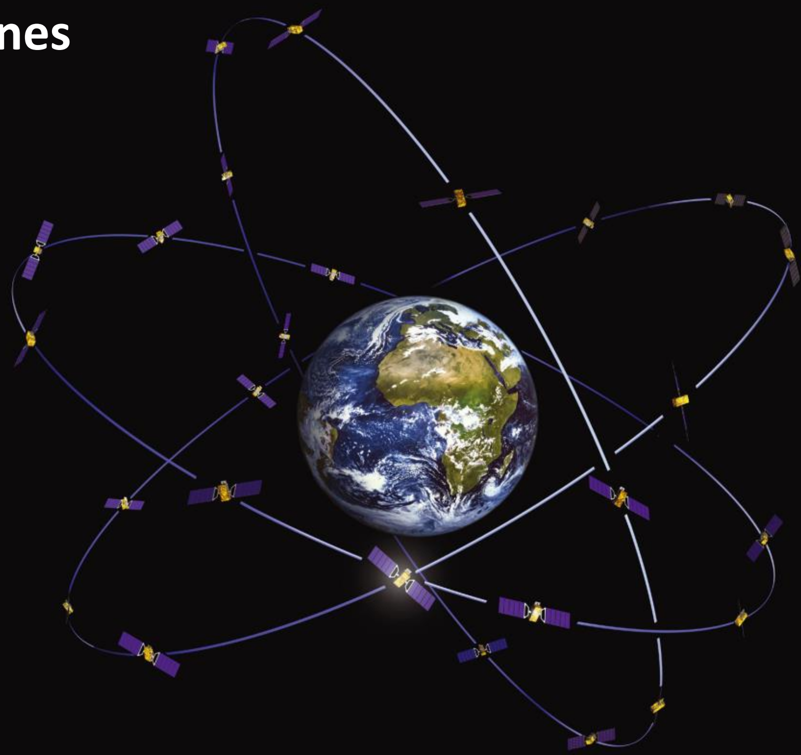


Visibility of GPS Satelites

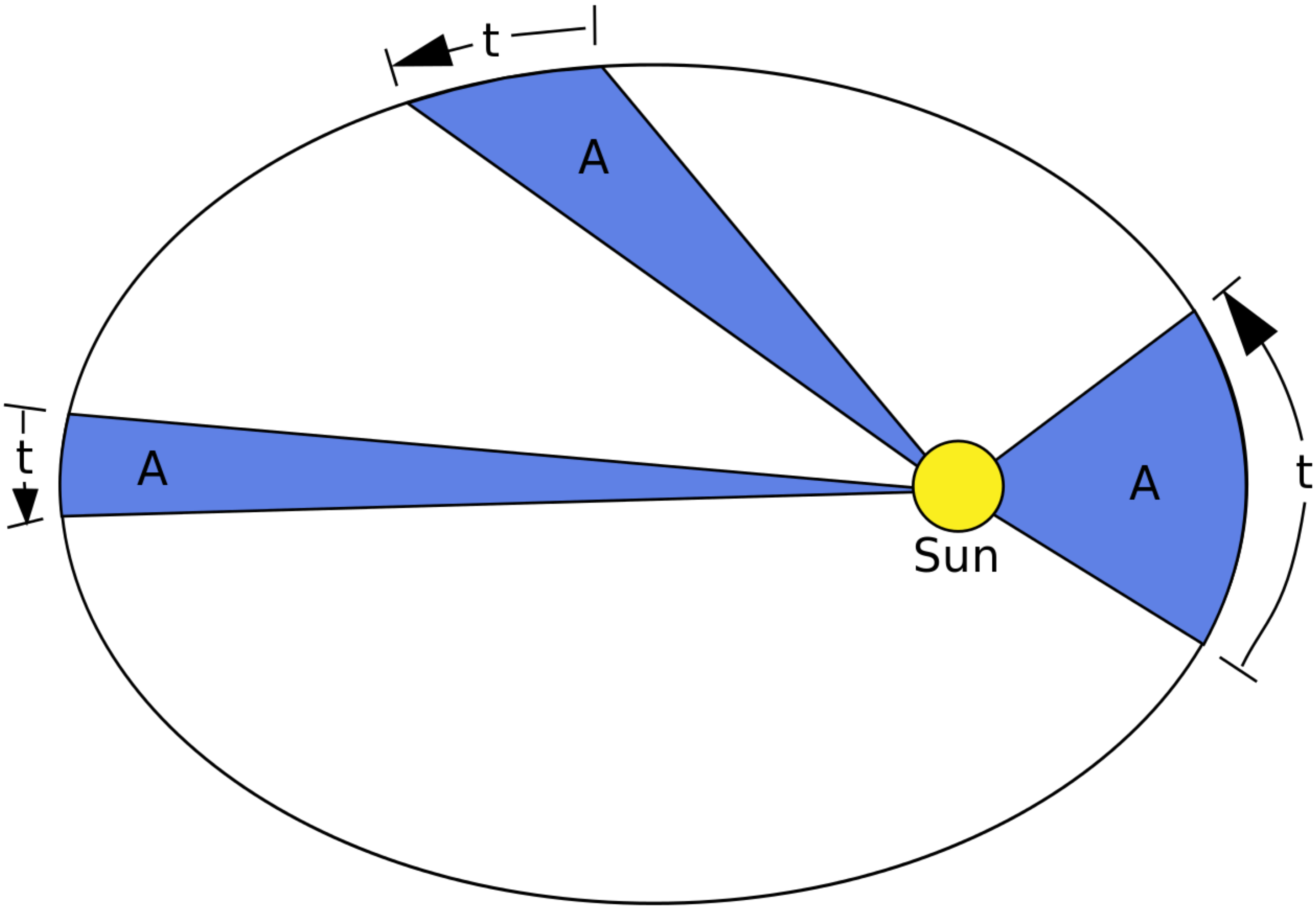


GALILEO Constellation

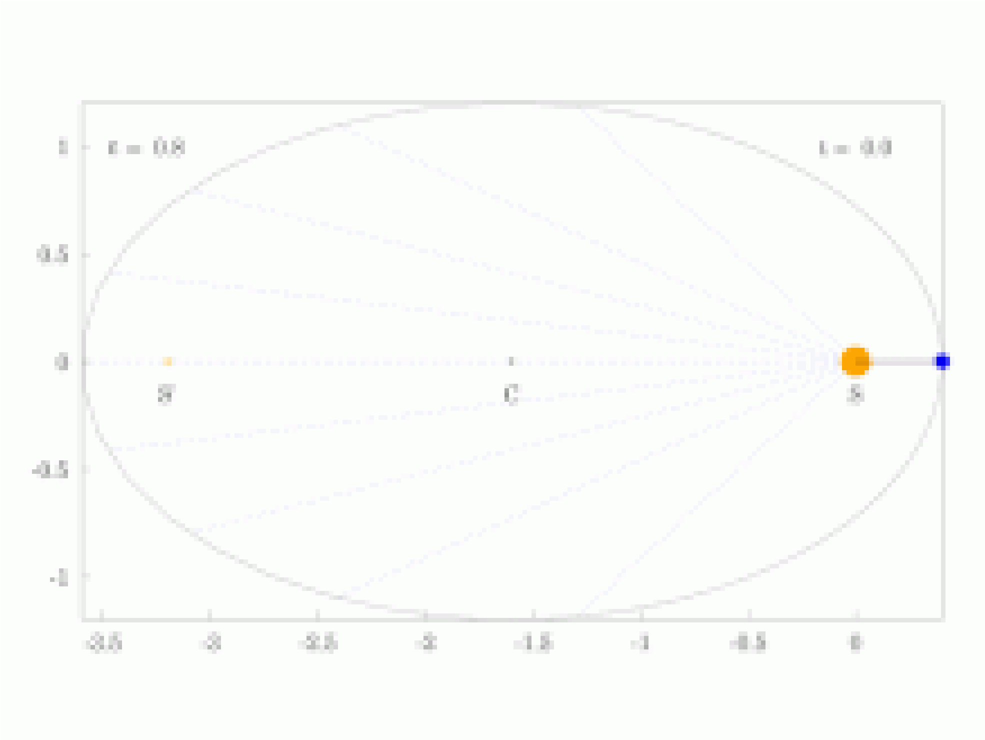
- 24 satellites (+6 spares)
- Circular Orbits on 3 equi-spaced planes
- Inclination of 56 degrees
- Altitude *h*=23,222 km
- Period (about)14 hours
- Better coverage of high-latitudes than GPS



Second Kepler Law



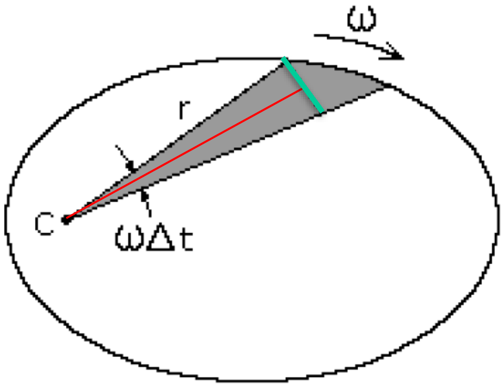
Second Kepler Law



Second Law, revisited

- The vector from the sun to the planet sweeps equal areas in equal times (the law of areas, 1605).

$$\Delta A = r \cdot \left(\frac{r \cdot \omega \Delta t}{2} \right) \quad \Rightarrow \quad \frac{dA}{dt} = \frac{\omega r^2}{2} = \text{constant}$$

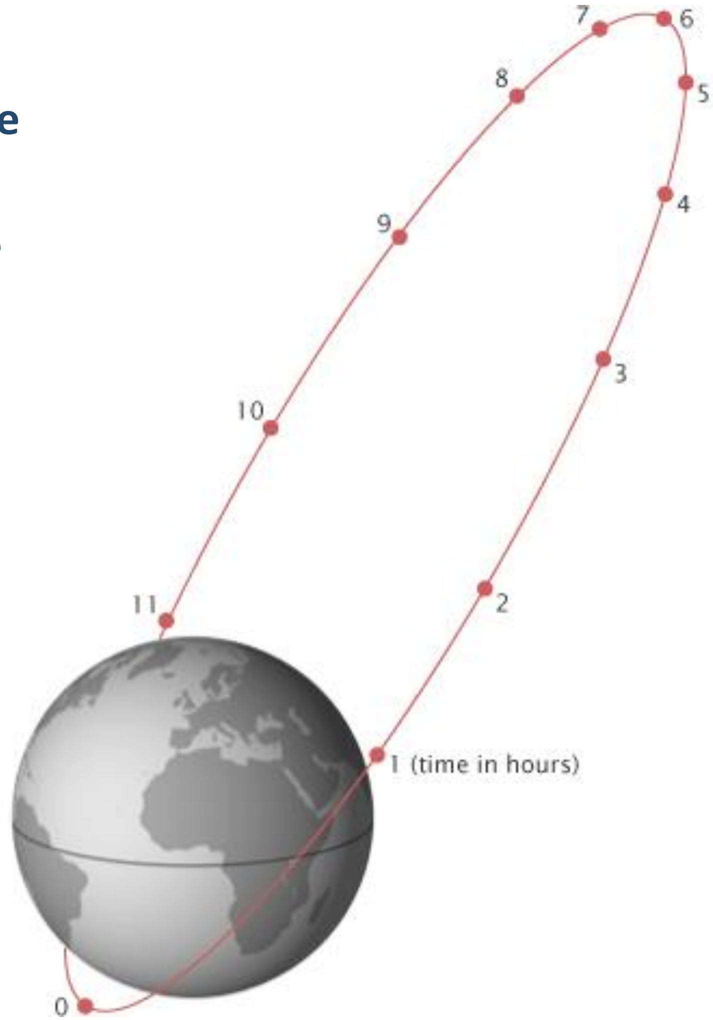


- So another formulation is: @any point

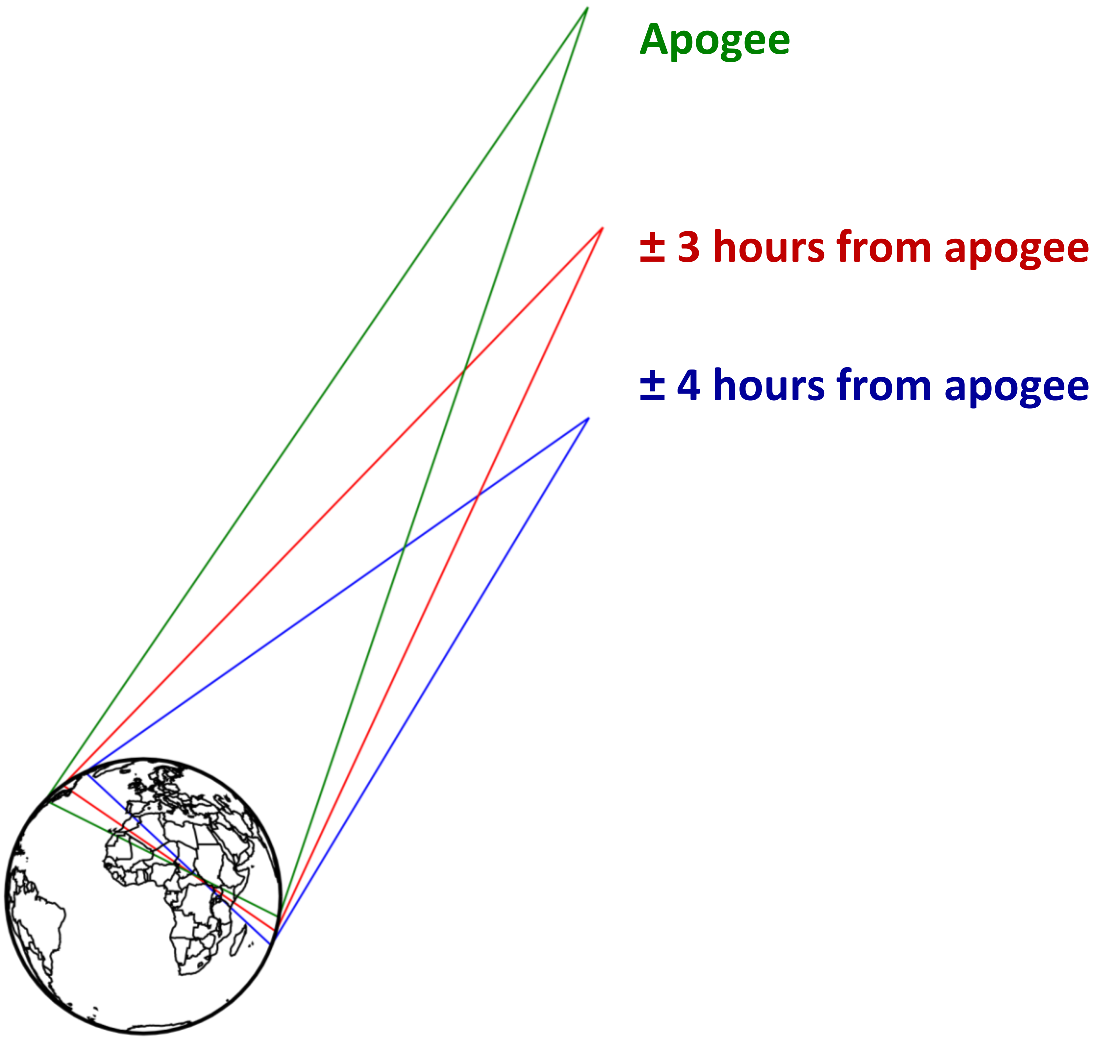
$$\omega r^2 = \text{constant}$$

Molniya Orbit

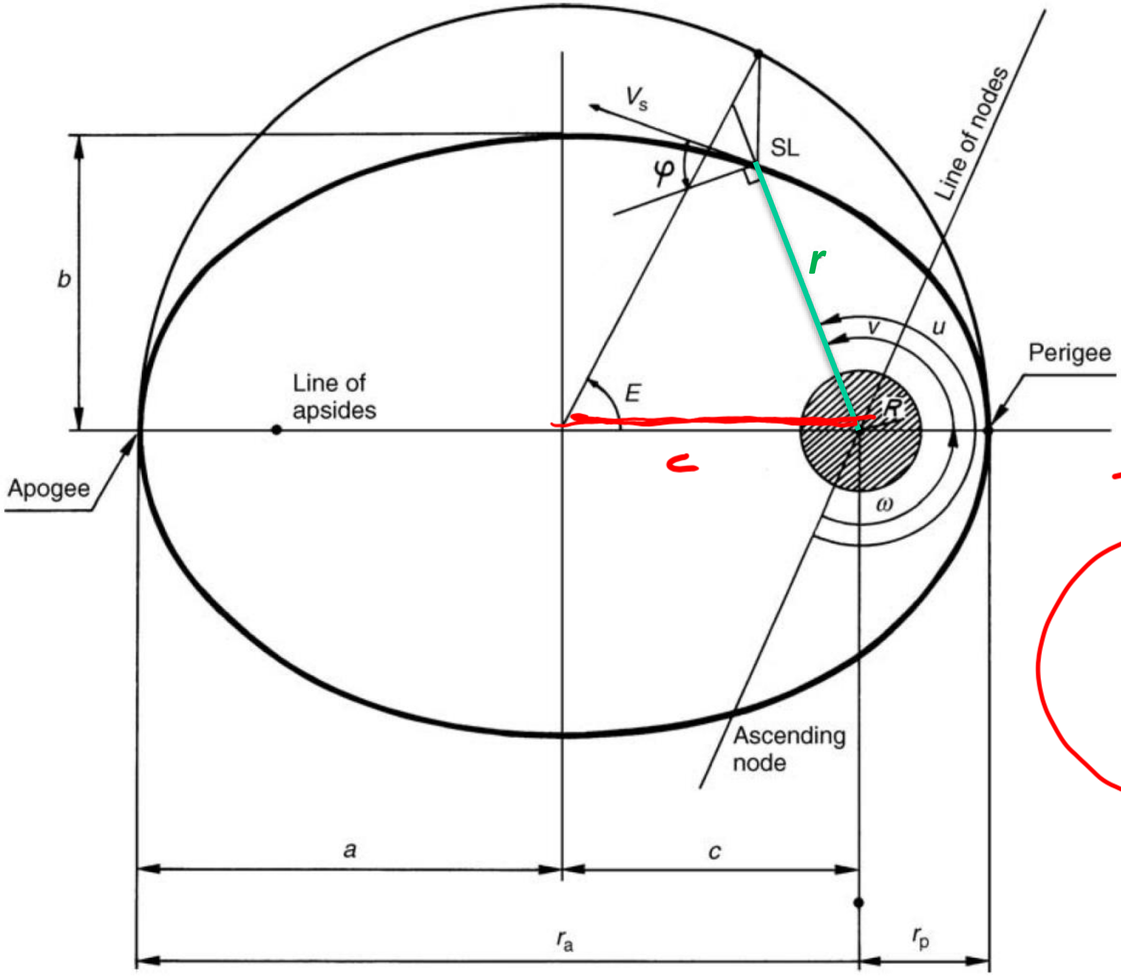
- **Russian Satellites to cover very high-latitude areas**
- **Tha satellite stays at apogee for a long time (2nd law…)**



Coverage of Molniya



Parameters of the Orbit



a*,*b semi-major and semi-minor ellipse axes

$c = (a^2 - b^2)^{1/2}$

r_a, r_p radius at apogee/perigee

$r_p = a - c$, $r_a = a + c$

$e = c/a$ eccentricity

$$e = \frac{\sqrt{a^2 - b^2}}{a} = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \frac{r_a - r_p}{r_a + r_p} = \frac{r_a - r_p}{2a}$$

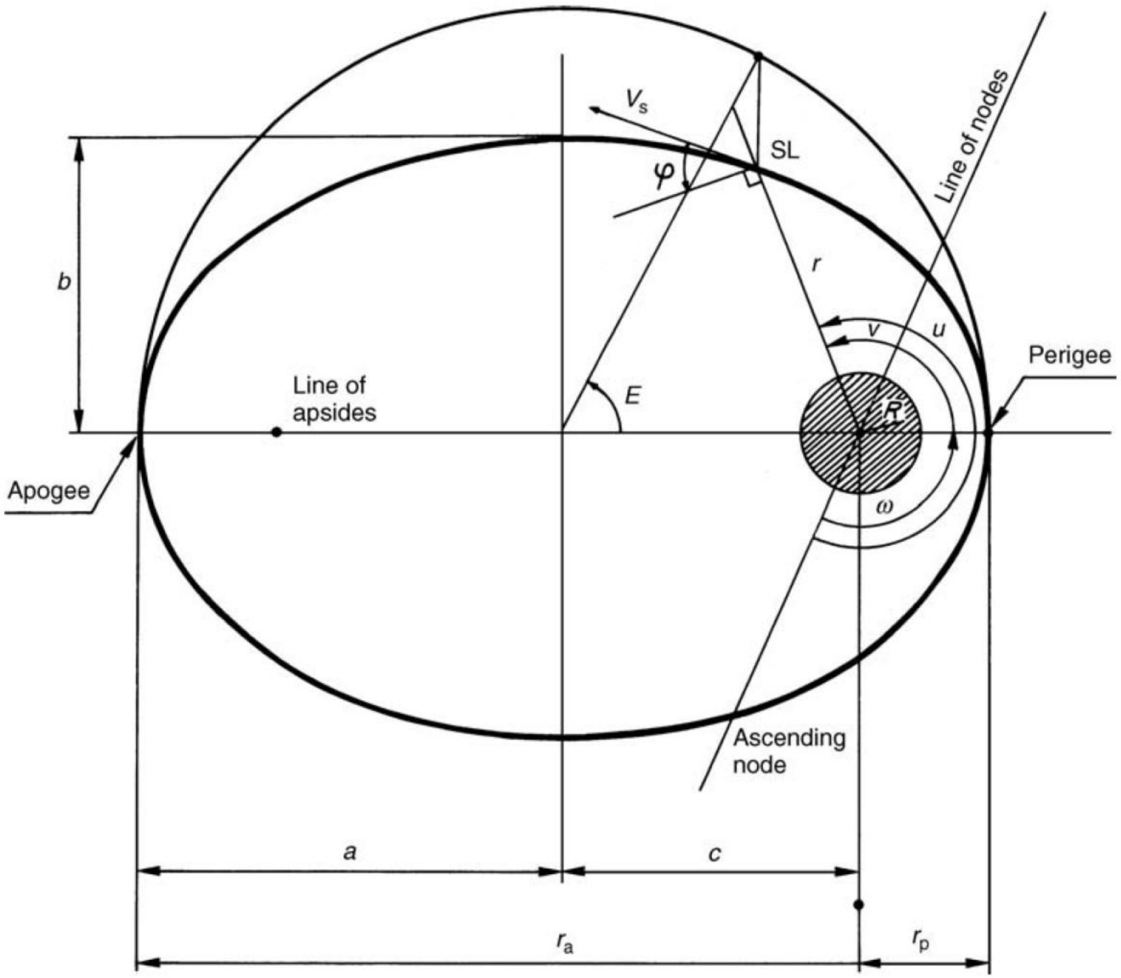
Orbits of Planets

Table 1. Orbital Data for the Planets

Planet	Semimajor Axis (AU)*	Period (y)	Eccentricity
Mercury	0.39	0.24	0.21
Venus	0.72	0.6	0.01
Earth	1	1.00	0.02
Mars	1.52	1.88	0.09
(**Ceres**)	2.77	4.6	0.08
Jupiter	5.20	11.86	0.05
Saturn	9.54	29.46	0.06
Uranus	19.19	84.01	0.05
Neptune	30.06	164.82	0.01

* 1 au = 149,597,870.707 km

Parameters of the Orbit



For any point on the elliptical orbit

$$r = a\frac{1-e^2}{1+e\cos(\nu)}$$

And the speed of the satellite is

$$v = \sqrt{GM\left(\frac{2}{r}-\frac{1}{a}\right)}$$

GM=μ **standard gravitational parameter 3.986 10^{14} m^3/s^2**

Dip. Ingegneria dell'Informazione
University of Pisa, Italy

Computation of the Satellite Speed 1/2

- Conservation of energy (kinetic+gravitational potential)

$$\frac{mv^2}{2} + \left(-\frac{GMm}{r} \right) = \text{constant}$$

- @ perigee/apogee, + conservation of angular momentum ***r*** x *m**v*** (***r*** $\perp$ ***v***)

$$\frac{v_a^{\,2}}{2} + \left(-\frac{GM}{r_a} \right) = \frac{v_p^{\,2}}{2} + \left(-\frac{GM}{r_p} \right) \qquad mv_a r_a = mv_p r_p \quad \Rightarrow \quad v_p = v_a \cdot r_a / r_p$$

- So

$$\frac{v_a^{\,2}}{2} \left(1 - \frac{r_a^{\,2}}{r_p^{\,2}} \right) = GM \left(\frac{1}{r_a} - \frac{1}{r_p} \right) \quad \Rightarrow \quad \frac{v_a^{\,2}}{2} = GM \, \frac{r_p}{r_a} \frac{1}{(r_p + r_a)}$$

Computation of the Satellite Velocity 2/2

- But

$$r_p + r_a = 2a \quad \Rightarrow \quad \frac{v_a^{\,2}}{2} = GM\,\frac{r_p}{r_a}\,\frac{1}{2a} = GM\,\frac{2a - r_a}{2a \cdot r_a} = GM\left(\frac{1}{r_a} - \frac{1}{2a}\right)$$

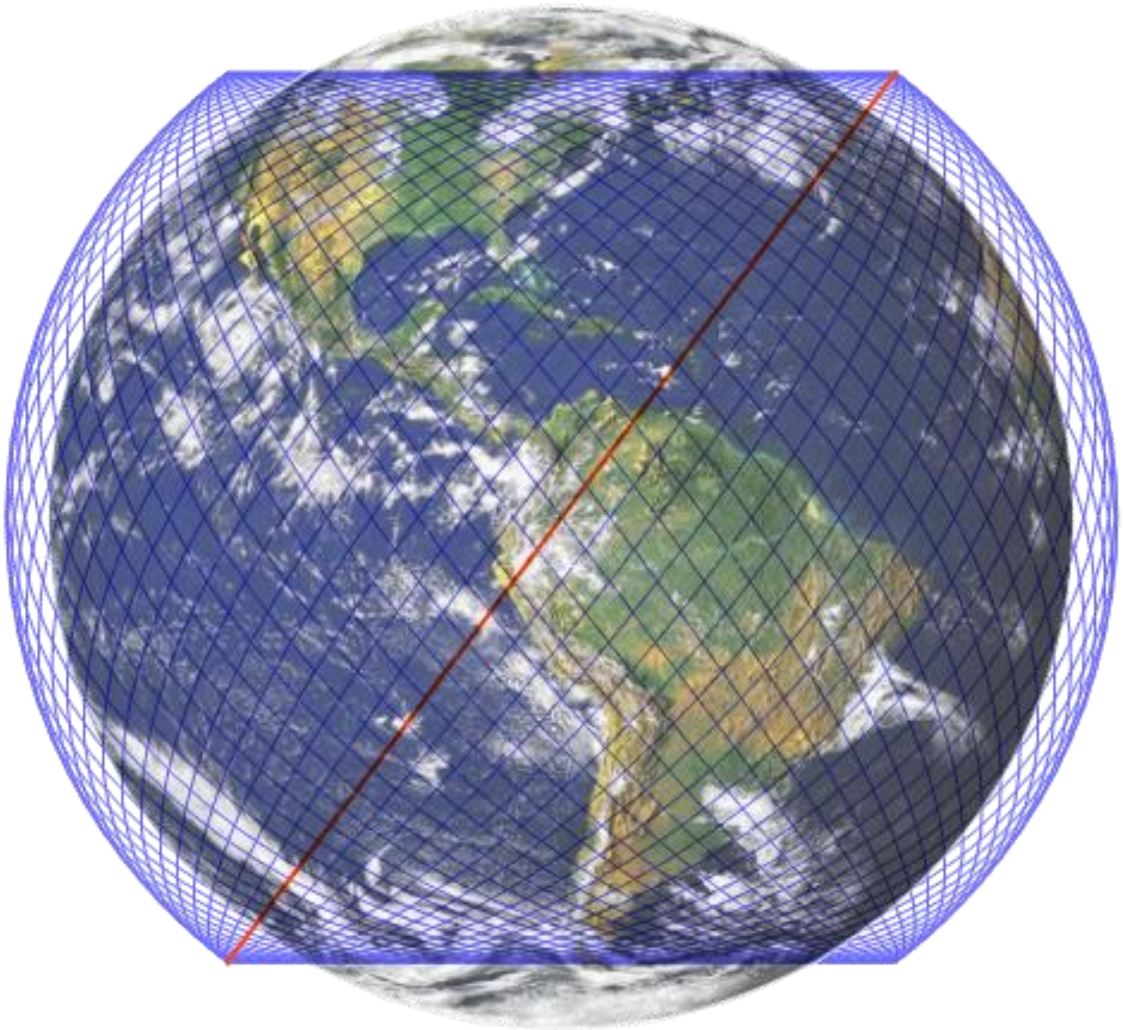
- so we can generalize and get back to the start: for any point

$$\frac{v^2}{2} = GM\left(\frac{1}{r} - \frac{1}{2a}\right) \quad \Rightarrow \quad v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$$

- For circular obits ($r=a$, $v=(GM/r)^{1/2}$)

Altitude (km)	Radius (km)	Period (s)	Velocity ($\mathrm{m\,s^{-1}}$)
200	6578	5309	7784
290	6668	5419	7732
800	7178	6052	7450
20000	26378	42636	3887
35786	42164	86164	3075

Megaconstellations: StarLink



About 3,000 satellites as of Sept. 2022, out of a planned total of FCC-approved 12,000

Nominal altitude: 550 km

Inclination: 53 degrees

Megaconstellations: OneWeb

Planned total of 684 small satellites

Nominal altitude: 1200 km

Inclination: 86.4 degrees (quasi-polar)

As for the Third Law…

- The ratio of the square of the period *T* of revolution of a planet around the sun to the cube of the semi-major axis *a* of the ellipse is the same for all planets (1618).

$$T^2 = \alpha \cdot a^3$$

- For the GEO we found

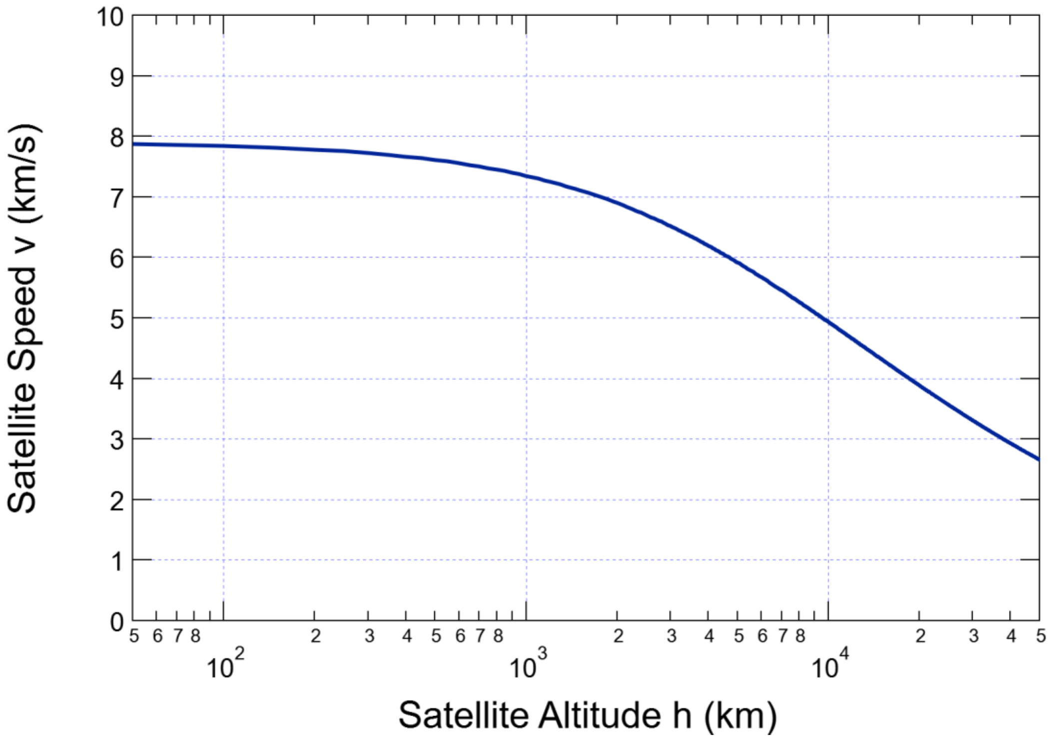
$$G\frac{mM}{r^2} = m\omega^2 r \quad \Rightarrow \quad \frac{GM}{a^3} = \frac{4\pi^2}{T^2}$$

- Therefore, we can find α from the GEO special case:

$$\alpha = \frac{4\pi^2}{GM} \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{a^3}{GM}}$$

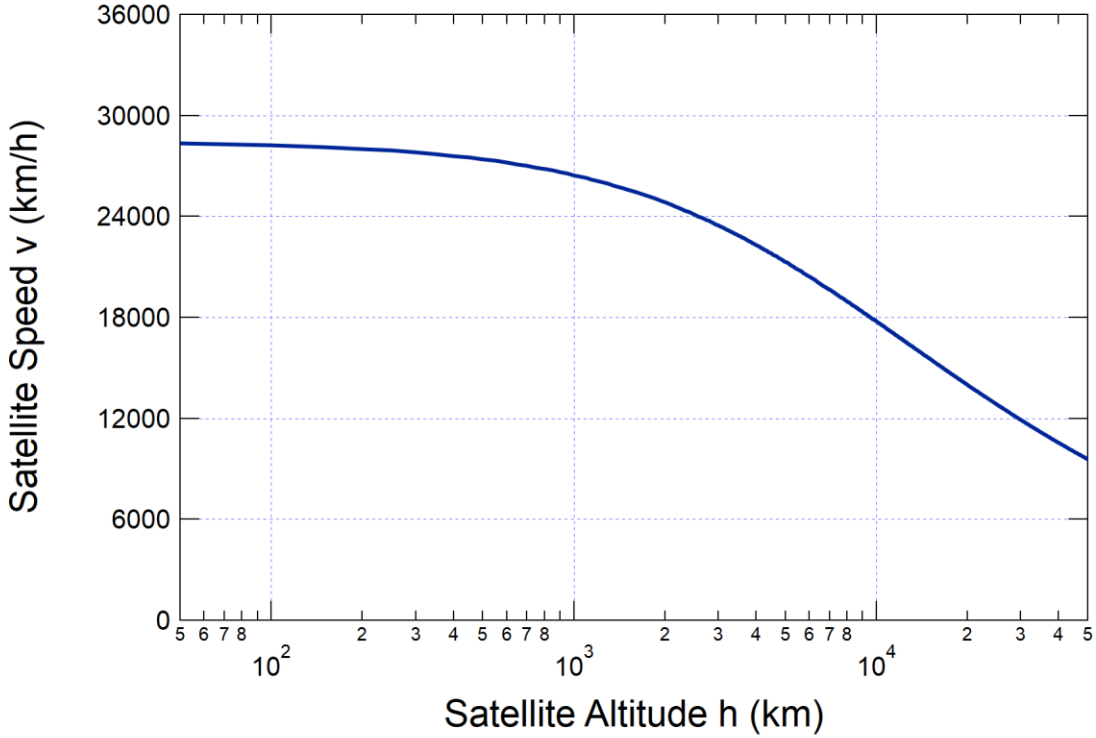
As for the Third Law (circular orbit)…

$$a = r = R + h \qquad T = 2\pi\sqrt{\frac{(R+h)^3}{GM}} \qquad v = \sqrt{\frac{GM}{(R+h)}}$$



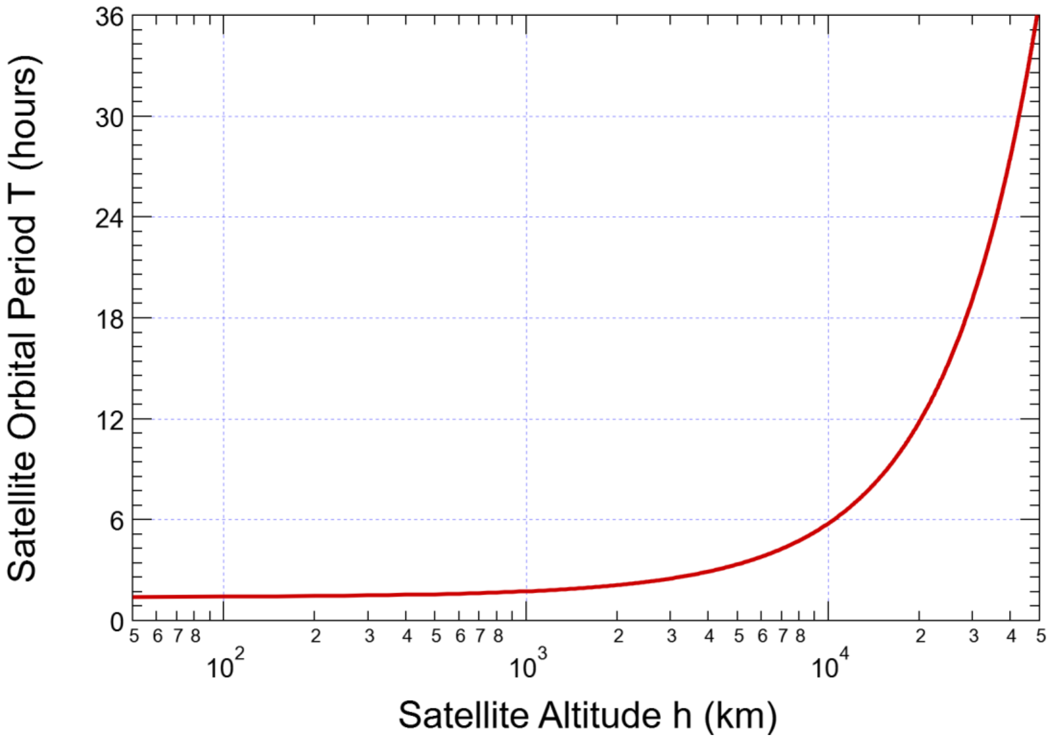
As for the Third Law (circular orbit)…

$$a = r = R + h \qquad T = 2\pi\sqrt{\frac{(R+h)^3}{GM}} \qquad v = \sqrt{\frac{GM}{(R+h)}}$$



As for the Third Law (circular orbit)…

$$a = r = R + h \qquad T = 2\pi\sqrt{\frac{(R+h)^3}{GM}} \qquad v = \sqrt{\frac{GM}{(R+h)}}$$



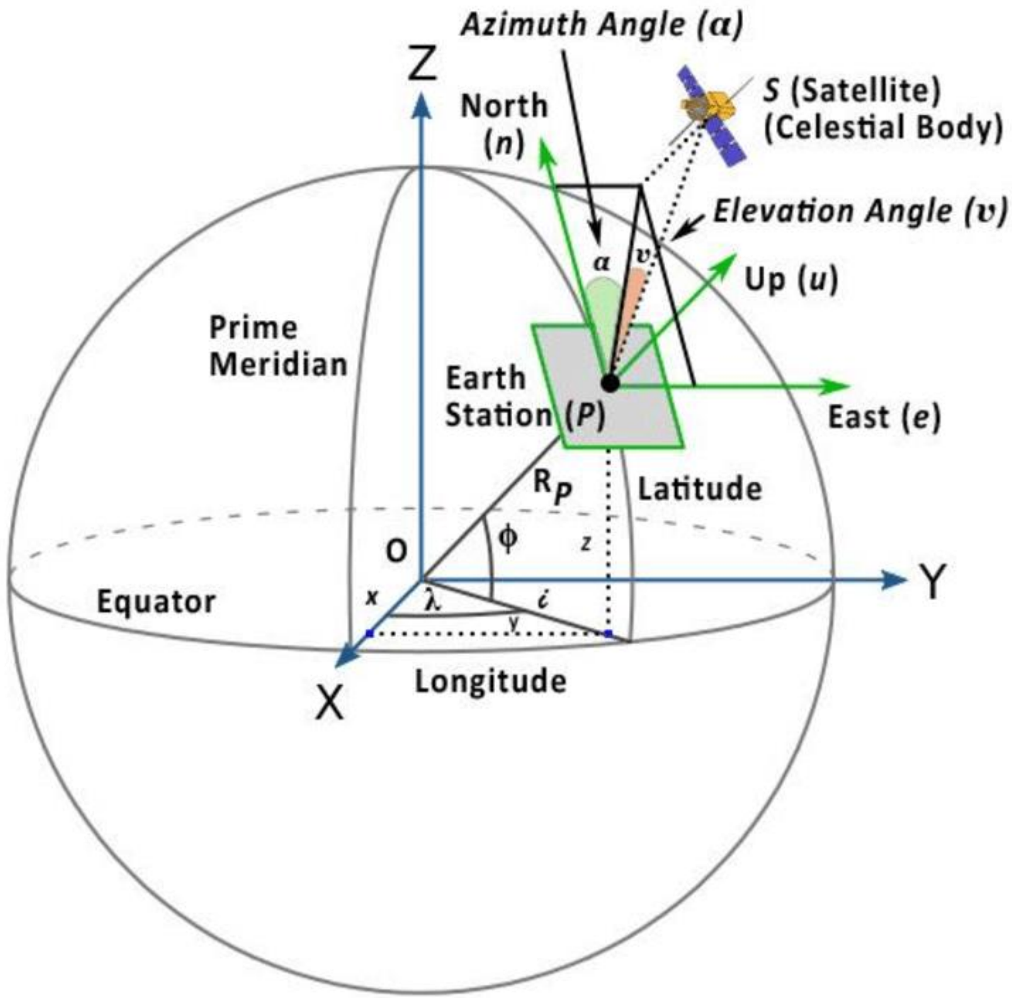
Coordinate Systems

- **Coordinates of the Earth Station (terminal)**
  - Left-handed Cartesian coordinate system *x,y,z* with O at the center of the Earth, the x-y plane coincident with the equatorial plane, and the x-axes lying in the polar plane containing Greenwich
  - Spherical coordinates system r,λ,ϕ with O at the center of the Earth, the *longitude* angle ϕ measured wrt the polar plane containing Greenwich, the *latitude* angle λ measured wrt the equatorial plane. Often, the *radius r* is decomposed as $r=R+h$ with h the *altitude* and R the Earth radius
  - Same reference and coordinates systems for the satellite, when needed

- **Coordinates of the Satellite wrt the Earth Station (ES)**
  - Define the plane tangent to the Earth surface @ the ES, two “local” *x-y* axes lying on such plane called North and East, the North axes pointing the North pole, and a “local” *z* axis called Up to make a left-handed system.
  - The “local longitude” is called *azimuth* α and the “local latitude” is called *elevation* ν (sometimes θ). The “local radius” is of course the *altitude h* as above

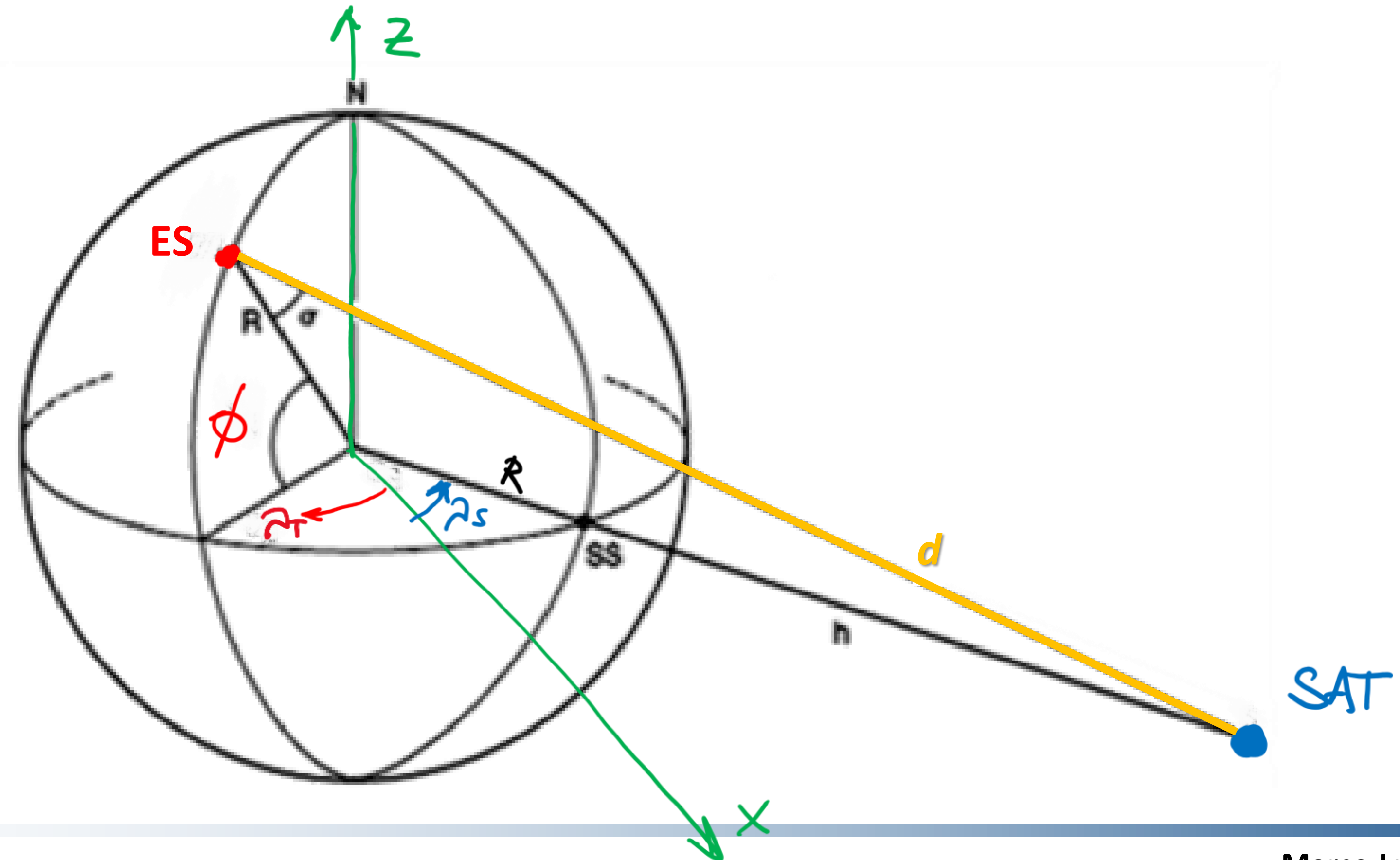
Dip. Ingegneria dell’Informazione
University of Pisa, Italy
Marco Luise

Satellite Angles

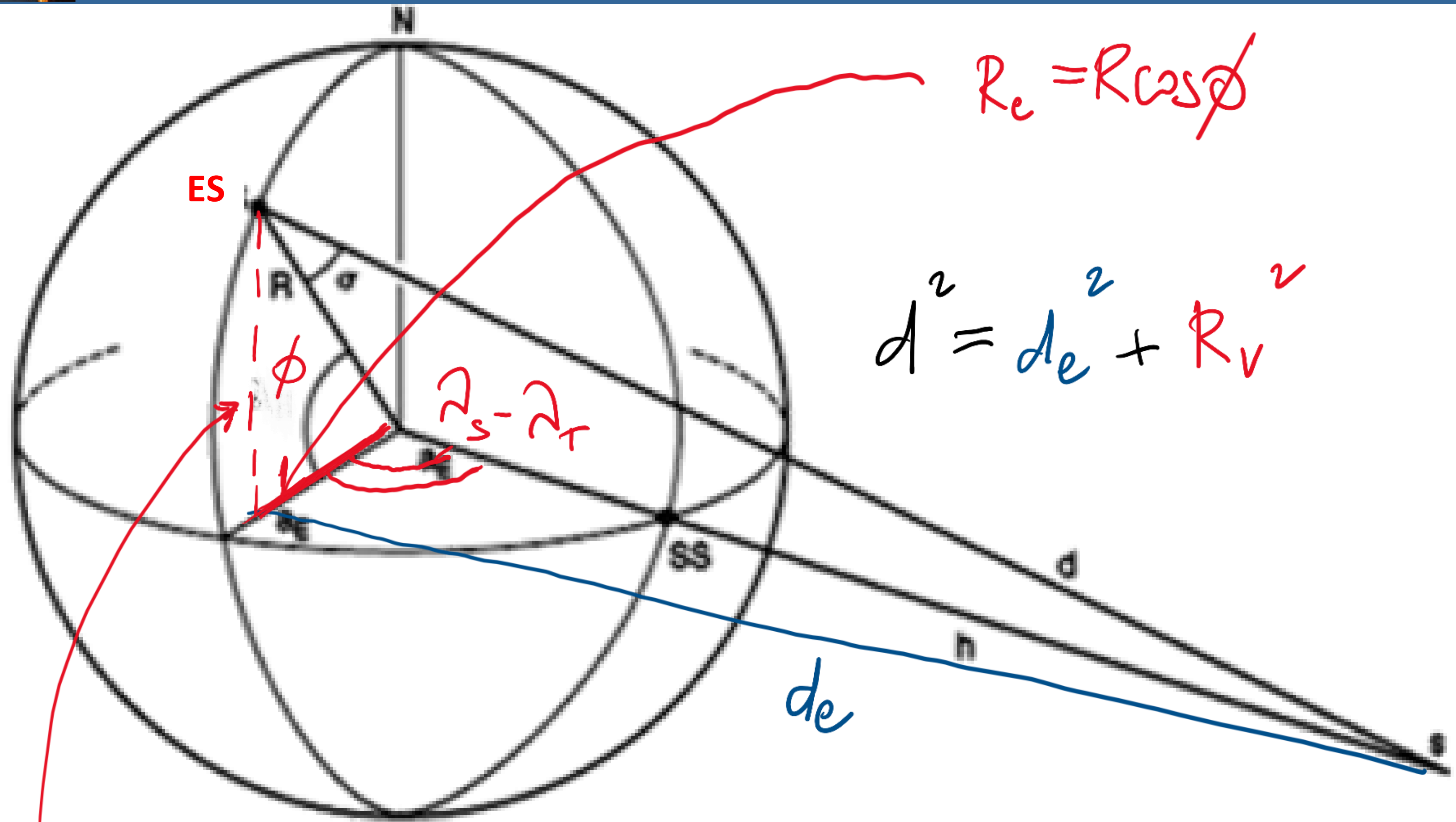


The Mother-of-All SatCom Orbits

- Satellite SAT latitude 0, longitude λ_S
- Earth Terminal ES latitude ϕ, longitude λ_T
- We need to compute the ES-SAT distance d



Computation of *d*



$$d^2 = d_e^2 + R_v^2$$

$$d_e^2 = (R+h)^2 + R_e^2 - 2R_e(R+h)\cos(\lambda_s - \lambda_T)$$

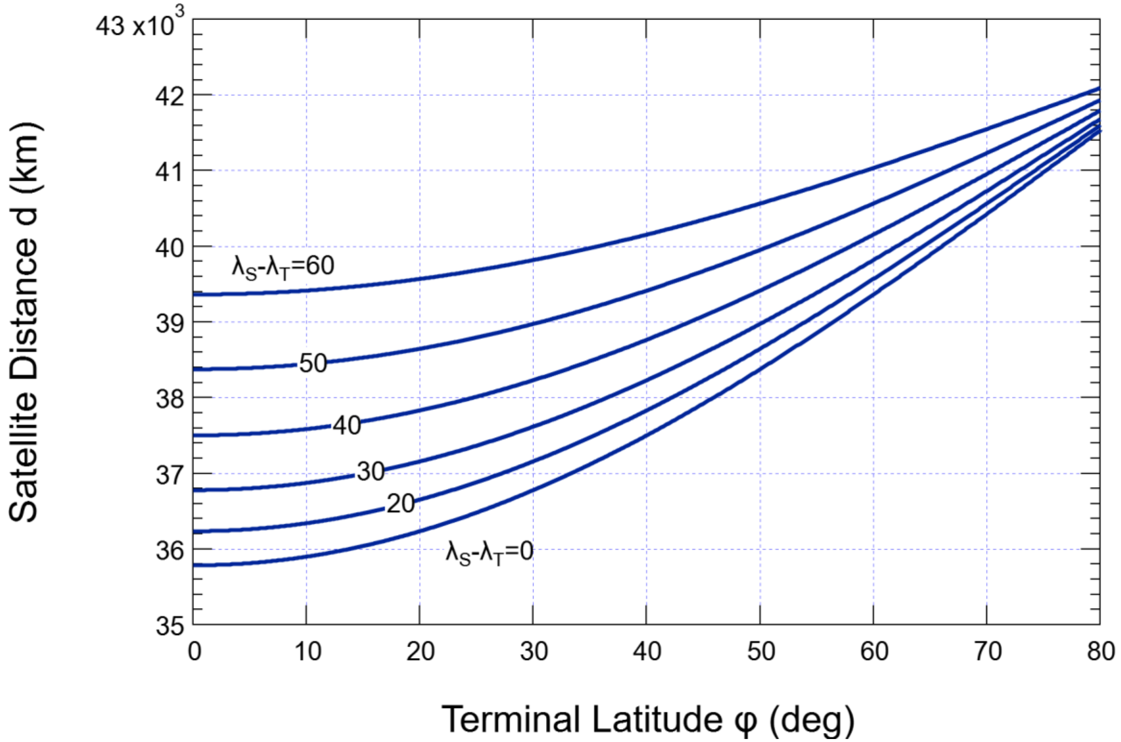
Dip. Ingegneria dell'Informazione
University of Pisa, Italy

Computation of *d*

$$d^2 = (R+h)^2 + R^2\cos^2(\phi) - 2(R+h)R\cos(\phi)\cos\left(\lambda_S - \lambda_T\right) + R^2\sin^2(\phi)$$

- **and finally**

$$d = \sqrt{(R+h)^2 + R^2 - 2(R+h)R\cos(\phi)\cos\left(\lambda_S - \lambda_T\right)}$$



To aim your dish: Elevation and Azimuth

- **Input:** your latitude λ_T and longitude ϕ, and the GEO SAT longitude λ_S
- **Output:** the elevation ν and azimuth α to mount your parabolic antenna

IT IS COMPLICATED

Elevation:

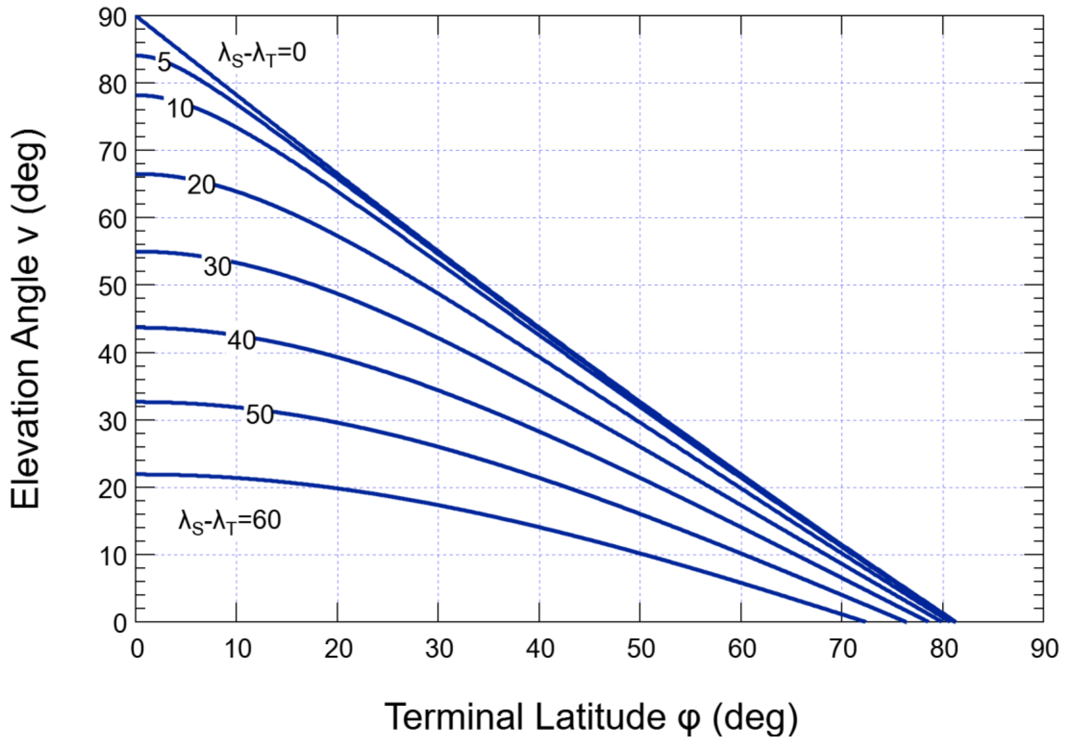
$$\nu = \arccos\left(\frac{R+h}{d}\right)\sqrt{1-\cos(\phi)^2\cos^2(\lambda_S-\lambda_T)}$$

Pseudo-Azimuth:

$$\alpha_0 = \arcsin\left(\frac{|\sin(\lambda_S-\lambda_T)|}{\sqrt{1-\cos(\phi)^2\cos^2(\lambda_S-\lambda_T)}}\right)$$

Elevation Chart

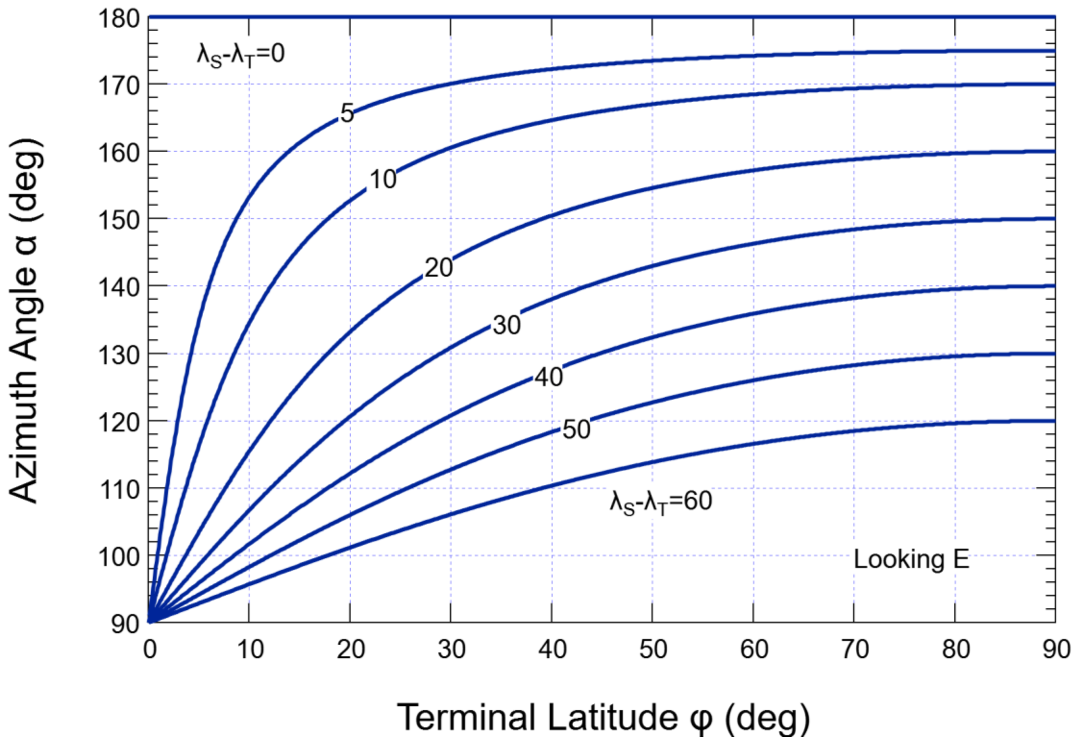
$$\nu = \arccos\left(\frac{R+h}{d}\right)\sqrt{1-\cos(\phi)^2\cos^2\left(\lambda_S-\lambda_T\right)}$$



Dip. Ingegneria dell'Informazione
University of Pisa, Italy
Marco Luise

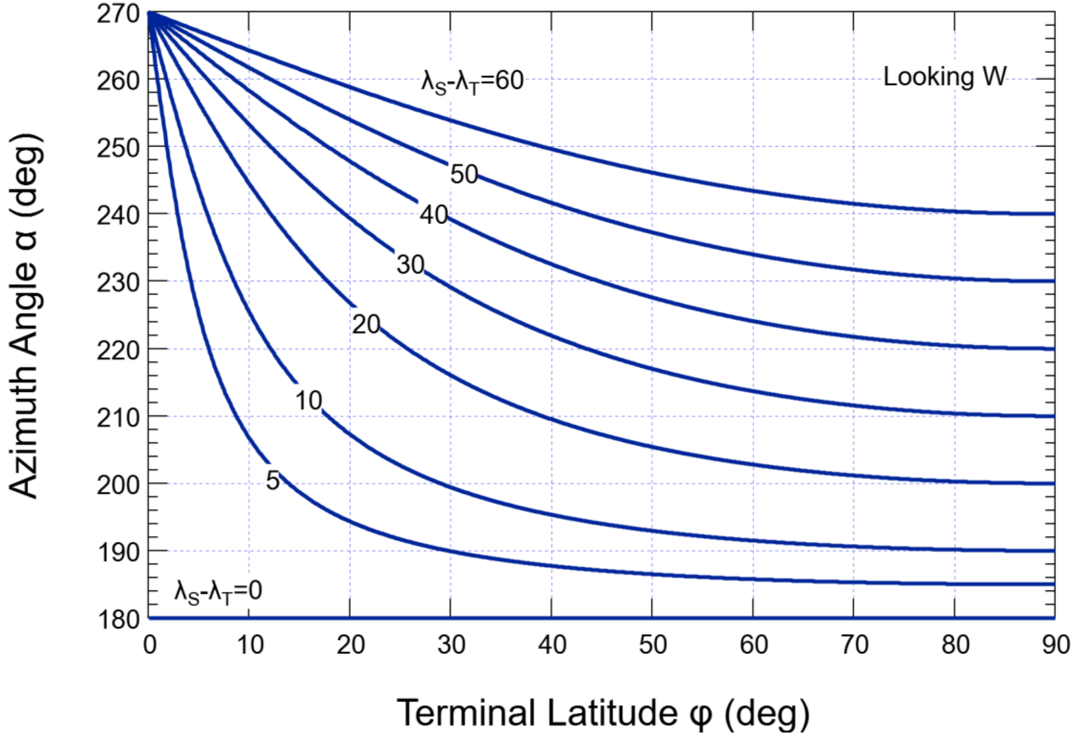
Azimuth Chart (Northern Hemisphere, looking EAST)

$$\alpha = 180 - \alpha_0 \quad \text{(deg)}$$



Azimuth Chart (Northern Hemisphere, looking WEST)

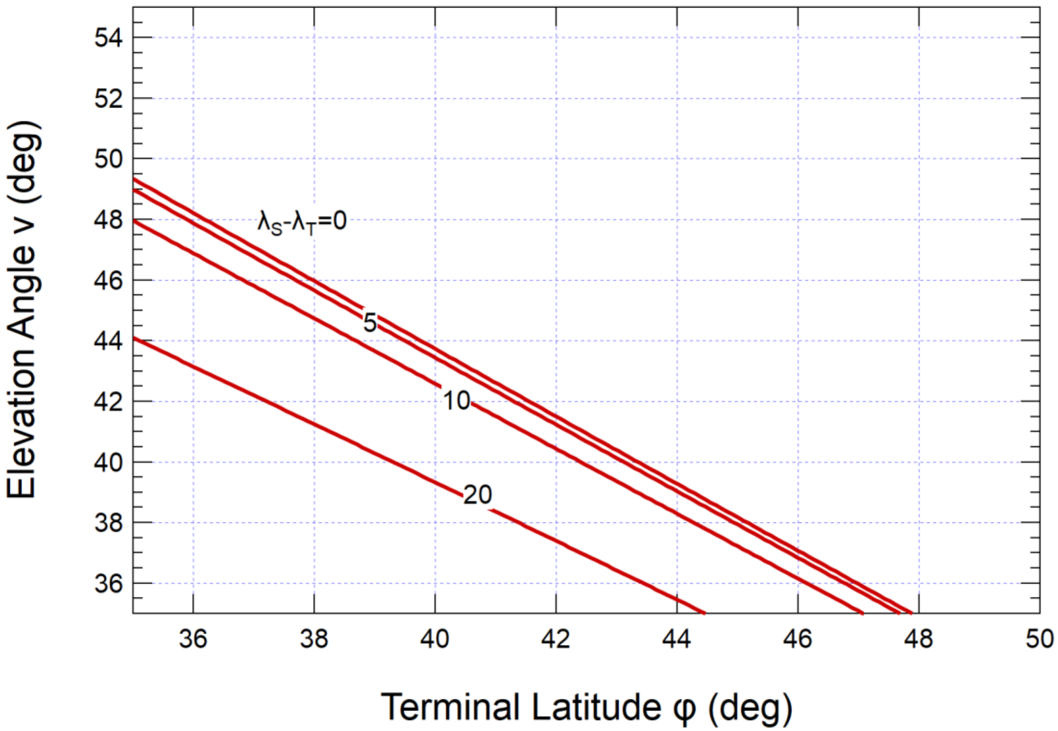
$$\alpha = 180 + \alpha_0 \quad \text{(deg)}$$

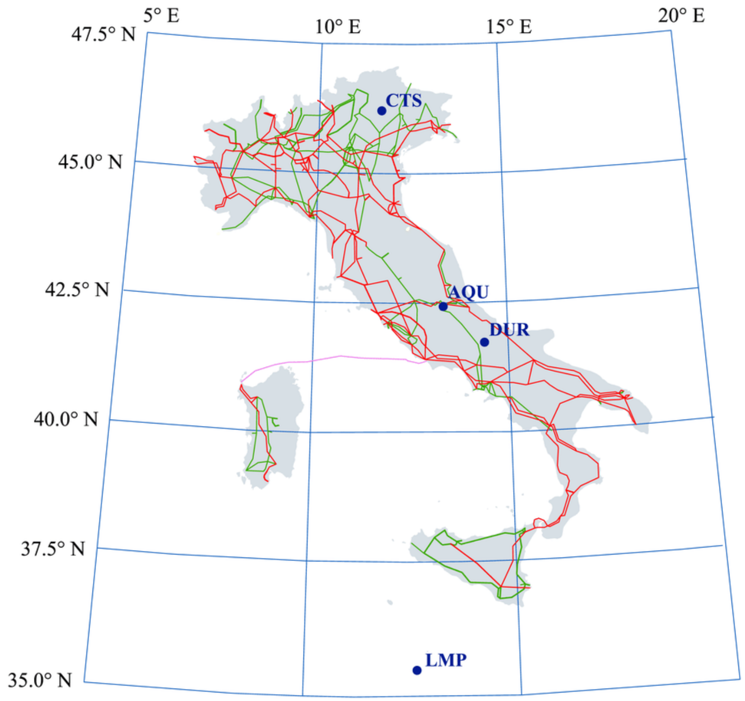


Try it out

- **Receiving SKY channels @ Pisa, Via Caruso (43.72N – 10.38E)**
  - In Italy, it is possible to receive free satellite channels of the Tivùsat platform which replicates the digital terrestrial offer (Rai Uno, Rai Due, Rai Tre, Canale 5, Italia 1, Rete 4, La 7), plus numerous other channels in HD and 4K.
  - Tivùsat transmits from the **HotBird** satellites of the Eutelsat fleet (13° East).
  - Sky channels are also broadcast through HotBird 13° East. For this reason, the vast majority of dishes in Italy are south-oriented, aiming at this satellite.
  - In Italy it is also possible to receive the channels broadcast by the SES Astra 19.2° Est satellite, which presents a rich offer of European and world television, with over 350 free channels (including Eurosport).

Elevation Chart for Italy

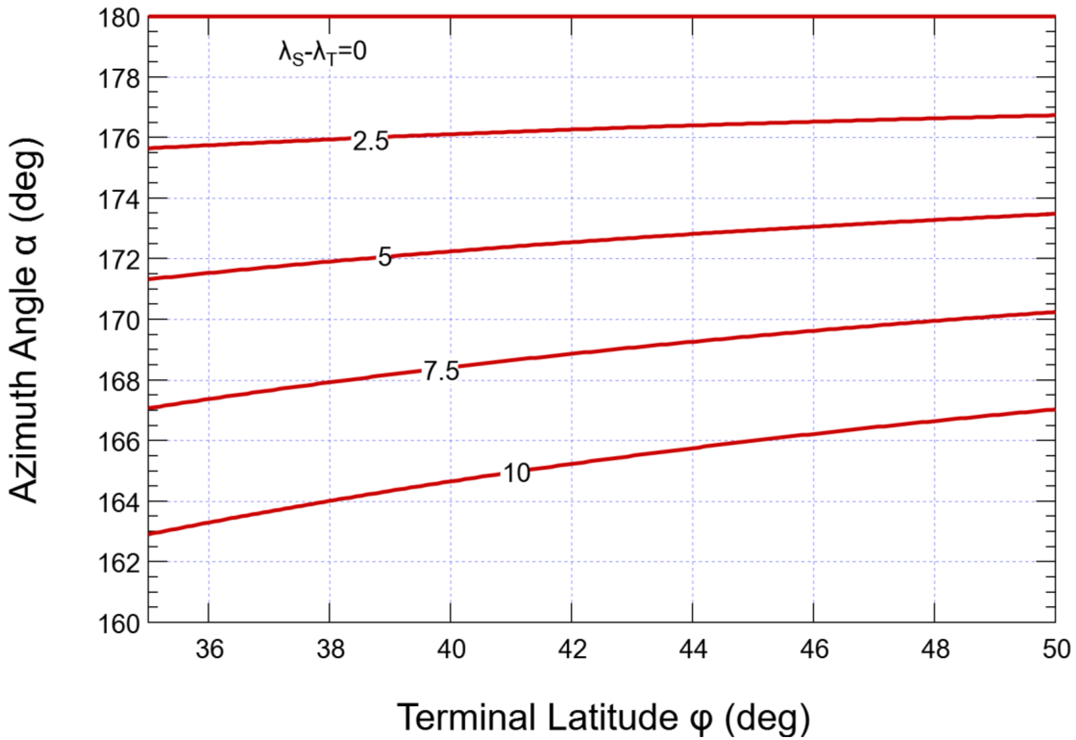


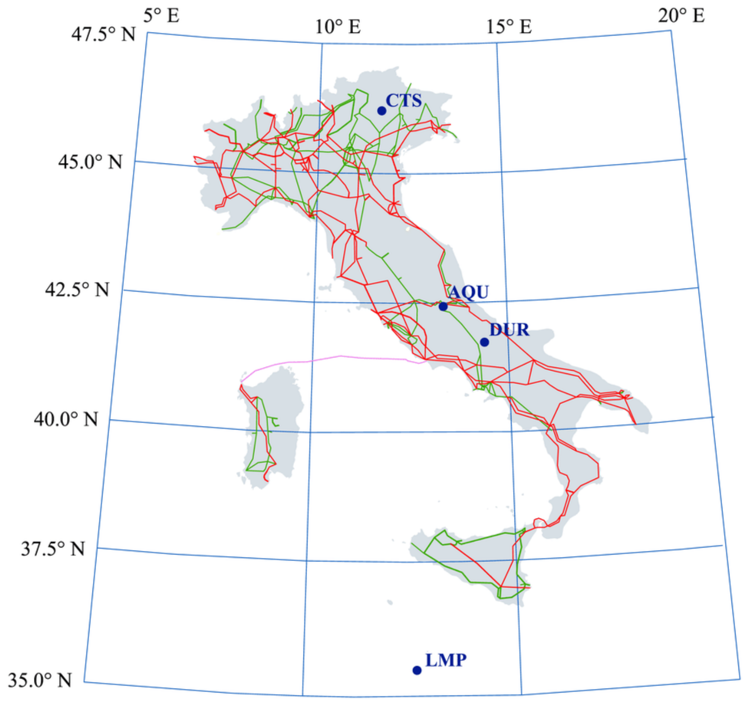


@DII for Hot Bird λ_S-λ_T =13-10.38=2.62 deg

@DII, ϕ=43.72 deg

Azimuth Chart for Italy, looking EAST





@DII for Hot Bird λ_S-λ_T =13-10.38=2.62 deg

@DII, ϕ=43.72 deg, α=180- α_0 deg


How do they launch it?
GALILEO
©2011 ESA - CNES - ARIANESPACE / Photo Optique Video CSG